Geometry

Medians of triangle



SECOND GRADE PREP .

First Term 2019

LESSON (1) Medians of triangle

Definition

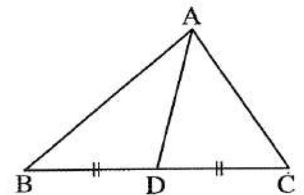
The median of a triangle is the line segment drawn from any vertex of this triangle to the midpoint of the opposite side of this vertex.

For example:

In the opposite figure :

If D is the midpoint of $\overline{BC}$

, then $\overline{AD}$ is a median of ΔABC



Notice that :

Any triangle has three medians.

Theorem 1

The medians of a triangle are concurrent.

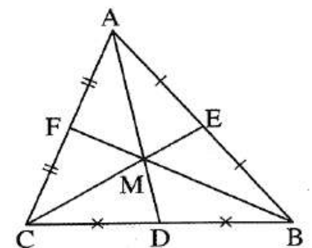
For example:

In the opposite figure :

$\overline{AD}$, $\overline{BF}$ and $\overline{CE}$ are the three medians of ΔABC ,

and they are concurrent at M

(**i.e.** $\overline{AD} \cap \overline{BF} \cap \overline{CE} = \{M\}$)



Mechanism : Point of Concurrence

Theorem 2

The point of concurrence of the medians of the triangle divides each median in the ratio of 1 : 2 from its base.

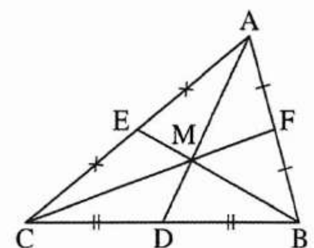
For example:

In the opposite figure :

In ΔABC , M is the point of concurrence of its medians , then :

1 $MD = \frac{1}{2} AM$ If AM = 6 cm. , then MD = 3 cm.

2 $CM = 2 FM$ If FM = 4 cm. , then CM = 8 cm.



Remark

The point of concurrence of the medians of the triangle divides each of them in the ratio of 2 : 1 from the vertex.

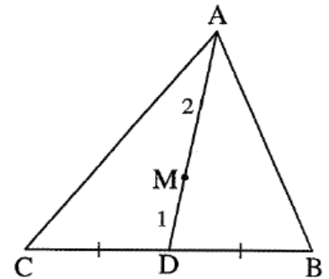
Fact

The point which divides the median in a triangle by the ratio of 1 : 2 from the base is the point of intersection of the medians of this triangle.

In the opposite figure :

If $\overline{AD}$ is a median in Δ ABC and M ∈ $\overline{AD}$ such that AM = 2 MD ,

then M is the point of intersection of the medians of Δ ABC



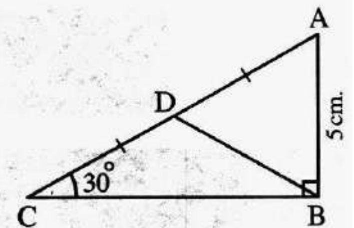
📖 Examples on Part : Medians Of Triangle 📖

① **In the opposite figure :**

m (∠ B) = 90° and m (∠ C) = 30°

, AB = 5 cm,

Find the length of : $\overline{AC}$ and $\overline{BD}$



Solution

In Δ ABC

∵ m (∠B) = 90°

∵ m (∠C) = 30°

∴ AC = 2 AB = 2 × 5 = 10 cm

(First Req.)

∵ D is a midpoint of $\overline{AC}$

∴ $\overline{BD}$ is a median

∴ BD = $\frac{1}{2}$ AC = $\frac{1}{2}$ × 10 = 5 cm

(Second Req.)

② **In the opposite figure :**

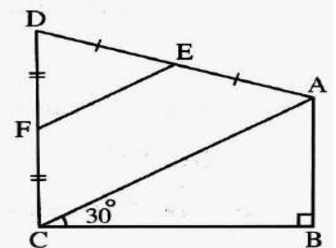
m (∠ B) = 90°

, m (∠ ACB) = 30°

, E is the midpoint of $\overline{AD}$

, F is the midpoint of $\overline{CD}$

Prove that : AB = EF



Solution

In Δ ABC

∵ m (∠B) = 90°

∵ E is a midpoint of $\overline{AD}$

∵ F is a midpoint of $\overline{DC}$

∵ m (∠C) = 30°

∴ BD = $\frac{1}{2}$ AC - - - - - - (1)

In Δ ADC

∴ EF = $\frac{1}{2}$ AC - - - - - - (2)

From (1) ad (2)

∴ AB = EF

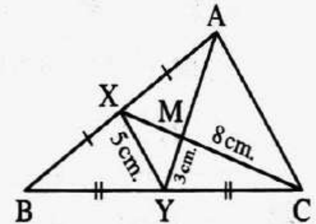
③ In the opposite figure :

ABC is a triangle , X is the midpoint of $\overline{AB}$

, Y is the midpoint of $\overline{BC}$, $\overline{XC} \cap \overline{AY}$ = {M}

, XY = 5 cm. , CM = 8 cm. , YM = 3 cm.

Find the perimeter of : Δ MAC



Solution

In Δ ABC

∵ Y is a midpoint of $\overline{BC}$

∵ X is a midpoint of $\overline{AB}$

∴ AC = 2 XY = 2 × 5 = 10 cm

∵ Y is a midpoint of $\overline{BC}$

∴ $\overline{AY}$ is a median In Δ ABC

∵ X is a midpoint of $\overline{AB}$

∴ $\overline{CX}$ is a median In Δ ABC

∴ M is the intersection point of medians In Δ ABC

∴ AM = 2 MY = 2 × 3 = 6 cm

The perimeter of Δ MAC =

6 + 8 + 10 = 24 cm

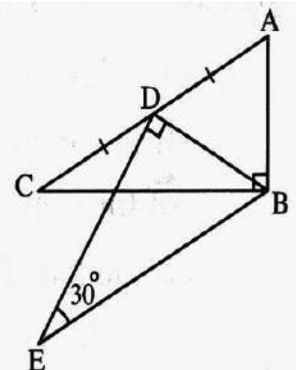
④ b] In the opposite figure :

m (∠ ABC) = m (∠ BDE) = 90°

, m (∠ E) = 30°

, D is the midpoint of $\overline{AC}$

Prove that : AC = BE



Solution

In Δ ABC

∵ m (∠B) = 90°

∵ D is a midpoint of $\overline{AC}$

In Δ DBE

∵ m (∠BDE) = 90°

∵ m (∠E) = 30°

∴ $\overline{BD}$ is a median

∴ BD = $\frac{1}{2}$ AC - - - - - - (1)

∴ BD = $\frac{1}{2}$ BE - - - - - - (2)

From (1) ad (2)

∴ AC = BE

⑤a] **In the opposite figure :**

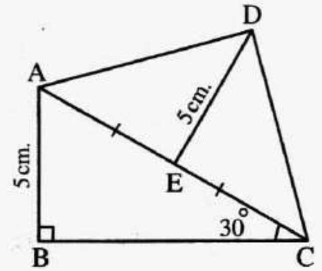
ABC is a right-angled triangle at B

, m (∠ ACB) = 30° , AB = 5 cm.

, E is the midpoint of $\overline{AC}$

If DE = 5 cm.

Prove that : m (∠ ADC) = 90°



Solution

In Δ ABC

∵ m (∠B) = 90°

∵ m (∠BCA) = 30°

∴ AB = $\frac{1}{2}$ AC

∵ AB = 5 cm

AC = 5 × 2 = 10 cm

In Δ ADC

∵ E is a midpoint of $\overline{AC}$

∴ $\overline{ED}$ is a median

∵ DE = 5 cm

∴ DE = $\frac{1}{2}$ AC

∴ m (∠ADC) = 90°

EXERCISES

[A] Complete the Following :

1	In Δ ABC : if the point X is the midpoint of $\overline{BC}$, then $\overline{AX}$ is called ....
2	The medians of the triangle are ....
3	The medians of the triangle intersect at ....
4	The point of intersection of the medians of a triangle divides each median in the ratio .... from the vertex.
5	The points of concurrence of the medians of the triangle divides each median in the ratio .... : .... from the base.
6	The point of intersection of the medians of the triangle divides each of them by the ratio 1 : 2 from ....
7	The point which divides the median of the triangle in the ratio 1 : 2 from the base is the point of ....

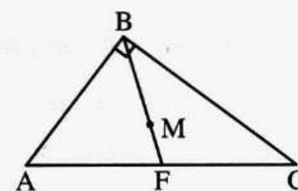
8

In the opposite figure :

If M is intersection point of medians

and m (∠ B) = 90° , MF = 1.5 cm.

, then the length of $\overline{AC}$ = ...



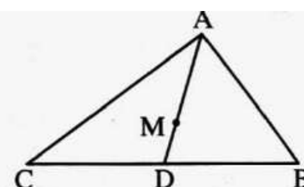
9

In Δ ABC : AB > AC , then m (∠ C) .... m (∠ B)

In the opposite figure :

If M is the point of intersection of

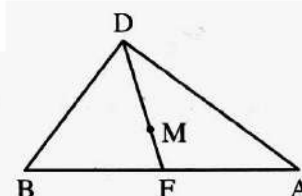
the medians of Δ ABC , then AM = ... AD



10

In the opposite figure :

If : MF = 2 cm., then DF = ...



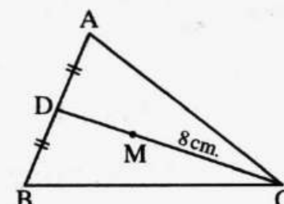
11

In the opposite figure :

In Δ ABC , M is the point of concurrence of the medians

, MC = 8 cm.

, then DM = ... cm.



12

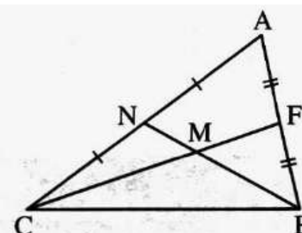
In the opposite figure :

If : F and N are the midpoints of $\overline{AB}$, $\overline{AC}$

Respectively , $\overline{BN} \cap \overline{CF} = \{m\}$, AB = 6 cm.

, AC = 10 cm. , BM = 4 cm. , CF = 9 cm.

Find the perimeter of figure : AFMN



[B] Essay Problems :

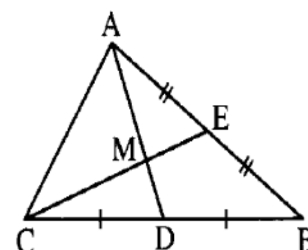
1

In the opposite figure :

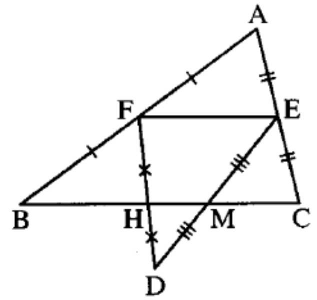
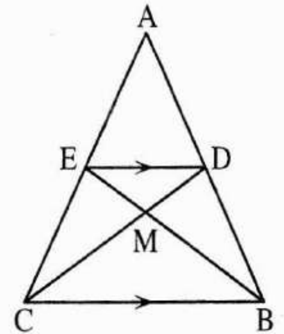
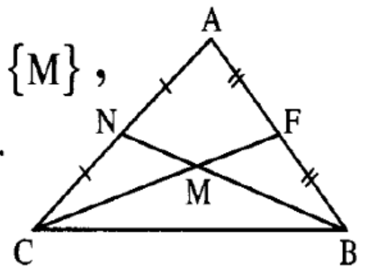
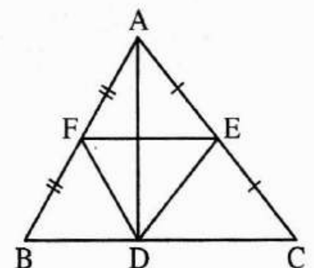
E is the midpoint of $\overline{AB}$, D is the midpoint of $\overline{BC}$

$\overline{AD} \cap \overline{CE} = \{M\}$, MC = 5 cm. and MD = 2 cm.

Find : The length of each of $\overline{AD}$ and $\overline{ME}$.



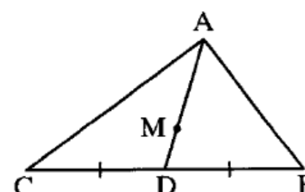
2015 Exam (1) Question (5) (a)

2	**In the opposite figure :** F , E , M and H are the midpoints of $\overline{AB}$, $\overline{AC}$, $\overline{ED}$ and $\overline{FD}$ respectively. **Prove that :** BC = 4 HM  **2015 Exam (10) Question (4) (a)**
3	**In the opposite figure :** ABC is a triangle in which $\overline{CD}$, $\overline{BE}$ two medians intersects at M , if : DC = 9 cm. , BM = 4 cm. , BC = 8 cm. **Find :** The perimeter of Δ MDE  **2012 Exam (15) Question (4) (b)**
4	**In the opposite figure :** F , N are midpoints of $\overline{AB}$, $\overline{AC}$ respectively , $\overline{BN} \cap \overline{CF} = \{M\}$, if : AB = 8 cm. , AC = 10 cm. , BM = 4 cm. and CF = 9 cm. **Find :** the perimeter of figure AFMN  **2015 Exam (6) Question (3) (a)**
5	**In the opposite figure :** ABC is a triangle , E , F are the midpoints of $\overline{AC}$ and $\overline{AB}$ respectively , $\overline{AD} \perp \overline{BC}$, AC = 18 cm. , BC = 20 cm. , AB = 16 cm. **Complete :** DF = ········ cm. , DE = ········ cm. , FE = ········ cm. , perimeter of Δ DEF = ········ cm.  **2012 Exam (17) Question (4) (a)**

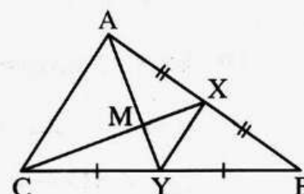
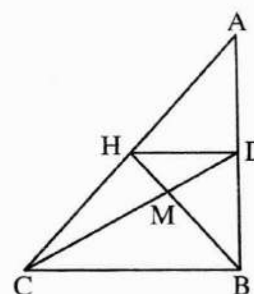
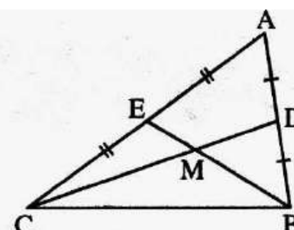
HOMEWORK

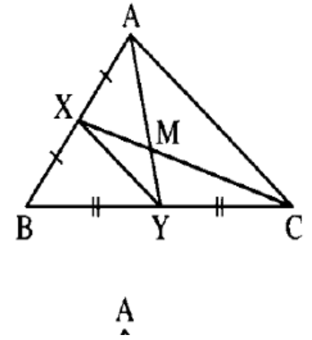
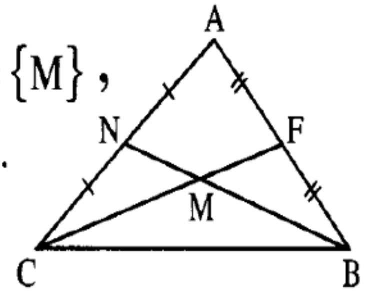
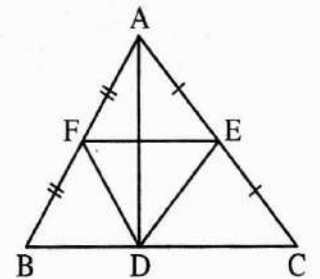
[A] Choose the correct answer:

1	The medians of the triangle intersect at ·········· point. (a) 1 (b) 2 (c) 3 (d) 4
2	The right-angled triangle has ·········· medians. (a) 0 (b) 1 (c) 2 (d) 3
3	The number of medians in the right-angled triangle = ··············· (a) 3 (b) 2 (c) 1 (d) 0
4	The point of intersection of the medians in the triangle divides each of them by the ratio ·········· from the vertex. (a) 1 : 3 (b) 3 : 1 (c) 2 : 1 (d) 1 : 2
5	The point of concurrence of the medians of the triangle divides each median in the ratio of ·········· from the base. (a) 1 : 2 (b) 1 : 3 (c) 2 : 1 (d) 3 : 1
6	If $\overline{AD}$ is a median of triangle ABC , and M is the point of intersection of the medians , then AM = ·········· AD (a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$
7	$\overline{AD}$ is a median in Δ ABC , M is the point of intersection of its medians , then AM = ·········· MD (a) 2 (b) $\frac{1}{2}$ (c) 3 (d) $\frac{1}{3}$
8	If $\overline{XE}$ is a median in Δ XYZ , M is the point of intersection of its medians , then EM = ·········· XE (a) $\frac{1}{2}$ (b) 2 (c) $\frac{1}{3}$ (d) $\frac{2}{3}$
9	In Δ ABC : If AD = 6 cm. is a median and M is a point of concurrent , then MA = ·········· cm. (a) 6 cm. (b) 3 cm. (c) 2 cm. (d) 4 cm.

10	If $\overline{AD}$ is a median of ΔABC , M is the point of intersection of its medians and AM = 6 cm. , then AD = ……… (a) 12 cm. (b) 6 cm. (c) 18 cm. (d) 9 cm.
11	oose the correct answer : **In the opposite figure :** $\overline{AD}$ is a median in ΔABC , M is the point of intersection of the medians , MD = 2 cm. , then AD = …………… cm. (a) 2 (b) 4 (c) 6 (d) 8 

[B] Essay Problems :

1	**In the opposite figure :** ABC is a triangle, X bisects $\overline{AB}$, Y bisects $\overline{BC}$, XY = 5 cm. , $\overline{XC} \cap \overline{AY} = \{M\}$ where CM = 8 cm. , YM = 3 cm. **Find with proof the length of :** $\overline{AC}$, $\overline{MX}$, $\overline{AM}$  **2014 Exam (13) Question (4) (a)**
2	**In the opposite figure :** ABC is a triangle in which $\overline{CD}$, $\overline{BH}$ are medians intersect at M , MC = 6 cm. , BC = 8 cm. , MB = 4 cm. **Find with proof :** The perimeter of ΔMDH  **2012 Exam (18) Question (3) (a)**
3	**In the opposite figure :** D and E are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively , $\overline{BE} \cap \overline{CD} = \{M\}$, If AB = 6 cm. , AC = 10 cm. , BM = 4 cm. and CD = 9 cm. **Find the perimeter of the figure :** ADME  **2014 Exam (14) Question (4) (b)**

4	**In the opposite figure :** X and Y are the midpoints of $\overline{AB}$ and $\overline{BC}$, $\overline{AY} \cap \overline{XC} = \{M\}$, if : AC = 14 cm. , XC = 15 cm. , and AM = 6 cm. **Find :** the perimeter of Δ XMY  **2015 Exam (2) Question (3) (a)**
5	**In the opposite figure :** F , N are midpoints of $\overline{AB}$, $\overline{AC}$ respectively , $\overline{BN} \cap \overline{CF} = \{M\}$, if : AB = 8 cm. , AC = 10 cm. , BM = 4 cm. and CF = 9 cm. **Find :** the perimeter of figure AFMN  **2015 Exam (6) Question (3) (a)**
6	**In the opposite figure :** ABC is a triangle , E , F are the midpoints of $\overline{AC}$ and $\overline{AB}$ respectively , $\overline{AD} \perp \overline{BC}$, AC = 18 cm. , BC = 20 cm. , AB = 16 cm. **Complete :** DF = ……… cm. , DE = ……… cm. , FE = ……… cm. , perimeter of Δ DEF = ……… cm.  **2012 Exam (17) Question (4) (a)**

LESSON (2) Medians of triangle (follow)

Mechanism : Median – Right-Angled Triangle :

Theorem 3

In the right-angled triangle , the length of the median from the vertex of the right angle equals half the length of the hypotenuse.

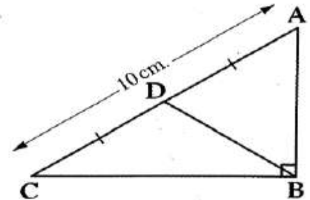
For example:

In the opposite figure :

Δ ABC is a right-angled triangle at B ,

D is the midpoint of $\overline{AC}$ and AC = 10 cm. ,

then DB = 5 cm.



Mechanism : Median To prove angle is right in Triangle

The converse of theorem 3

If the length of the median drawn from a vertex of a triangle equals half the length of the opposite side to this vertex, then the angle at this vertex is right.

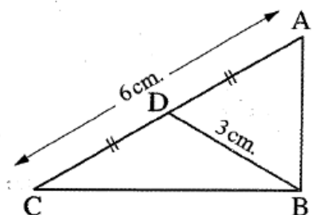
For example:

In the opposite figure :

If $\overline{BD}$ is a median in Δ ABC ,

BD = 3 cm. and AC = 6 cm. ,

then m ($\angle$ ABC) = 90° "because BD = $\frac{1}{2}$ AC"



Mechanism : Median – Right – 30° - 60°

Corollary

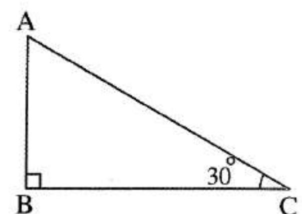
The length of the side opposite to the angle of measure 30° in the right-angled triangle equals half the length of the hypotenuse.

i.e.

In the opposite figure :

If Δ ABC is right-angled at B and

m ($\angle$ C) = 30° , then AB = $\frac{1}{2}$ AC



For example:

If AC = 20 cm. , then AB = 10 cm.

Remark

The right-angled triangle whose measures of angles are 30° , 60° and 90° is called thirty and sixty triangle.

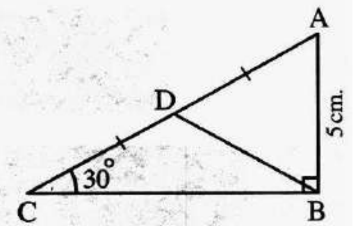
Examples on Part (1) : Medians Of Triangle

① **In the opposite figure :**

m (∠ B) = 90° and m (∠ C) = 30°

, AB = 5 cm,

Find the length of : $\overline{AC}$ and $\overline{BD}$



Solution

In Δ ABC

∵ m (∠B) = 90°

∵ m (∠C) = 30°

∴ AC = 2 AB = 2 × 5 = 10 cm

(First Req.)

∵ D is a midpoint of $\overline{AC}$

∴ $\overline{BD}$ is a median

∴ BD = $\frac{1}{2}$ AC = $\frac{1}{2}$ × 10 = 5 cm

(Second Req.)

② **In the opposite figure :**

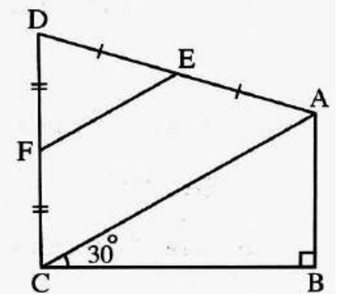
m (∠ B) = 90°

, m (∠ ACB) = 30°

, E is the midpoint of $\overline{AD}$

, F is the midpoint of $\overline{CD}$

Prove that : AB = EF



Solution

In Δ ABC

∵ m (∠B) = 90°

∵ m (∠C) = 30°

∴ BD = $\frac{1}{2}$ AC - - - - - - (1)

In Δ ADC

∵ E is a midpoint of $\overline{AD}$

∵ F is a midpoint of $\overline{DC}$

∴ EF = $\frac{1}{2}$ AC - - - - - - (2)

From (1) ad (2)

∴ AB = EF

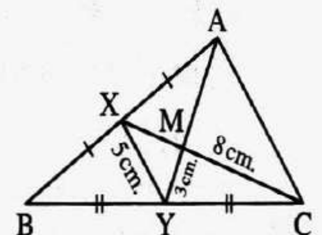
③ **In the opposite figure :**

ABC is a triangle , X is the midpoint of $\overline{AB}$

, Y is the midpoint of $\overline{BC}$, $\overleftrightarrow{XC} \cap \overleftrightarrow{AY}$ = {M}

, XY = 5 cm. , CM = 8 cm. , YM = 3 cm.

Find the perimeter of : Δ MAC

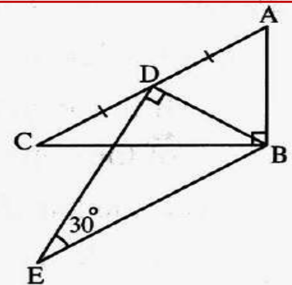


Solution

In Δ ABC
∵ Y is a midpoint of $\overline{BC}$
∵ X is a midpoint of $\overline{AB}$
∴ AC = 2 XY = 2 × 5 = 10 cm
∵ Y is a midpoint of $\overline{BC}$
∴ $\overline{AY}$ is a median In Δ ABC
∵ X is a midpoint of $\overline{AB}$
∴ $\overline{CX}$ is a median In Δ ABC
∴ M is the intersection point of medians In Δ ABC
∴ AM = 2 MY = 2 × 3 = 6 cm
The perimeter of Δ MAC =
6 + 8 + 10 = 24 cm

④ b] **In the opposite figure :**

m (∠ ABC) = m (∠ BDE) = 90°
, m (∠ E) = 30°
, D is the midpoint of $\overline{AC}$
Prove that : AC = BE

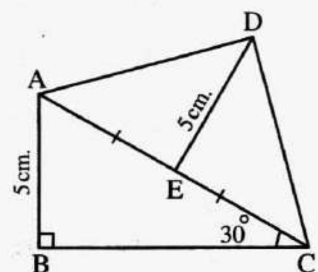


Solution

In Δ ABC
∵ m (∠B) = 90°
∵ D is a midpoint of $\overline{AC}$
In Δ DBE
∵ m (∠BDE) = 90°
∵ m (∠E) = 30°
∴ $\overline{BD}$ is a median
∴ BD = $\frac{1}{2}$ AC - - - - - - (1)
∴ BD = $\frac{1}{2}$ BE - - - - - - (2)
From (1) ad (2)
∴ AC = BE

⑤ a] **In the opposite figure :**

ABC is a right-angled triangle at B
, m (∠ ACB) = 30° , AB = 5 cm.
, E is the midpoint of $\overline{AC}$
If DE = 5 cm.
Prove that : m (∠ ADC) = 90°



Solution

In Δ ABC
∵ m (∠B) = 90°
∵ m (∠BCA) = 30°
∴ AB = $\frac{1}{2}$ AC
∵ AB = 5 cm
AC = 5 × 2 = 10 cm
In Δ ADC
∵ E is a midpoint of $\overline{AC}$
∴ $\overline{ED}$ is a median
∵ DE = 5 cm
∴ DE = $\frac{1}{2}$ AC
∴ m (∠ADC) = 90°

EXERCISES

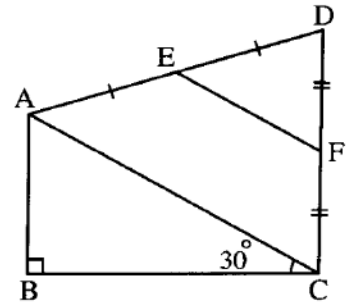
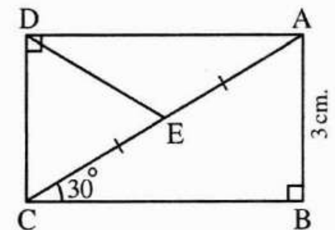
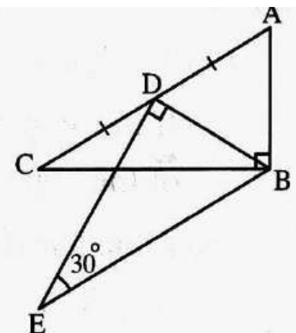
[A] Complete the Following :

1	In the right-angled triangle the length of the median from the vertex of the right angle equal ………. the length of the hypotenuse.
2	In the right-angled triangle , the length of the median from the vertex of the right angle equals ……….
3	If the length of the median drawn from a vertex of a triangle equals half the length of the opposite side to this vertex in length , then ……….
4	The length of the side opposite to the angle of measure 30° in the right-angled triangle equals ………. the length of the hypotenuse.
5	The length of side opposite to the angle whose measure = 30° in the right-angled triangle = ……….
6	The length of the hypotenuse on the right-angled triangle equals ………. the length of a side opposite to the angle of measure 30°
7	In Δ LMN : If m (∠ L) = 30° , m (∠ N) = 60° , NM = 4 cm. , then LN = ……… cm.
8	If ABC is a right-angled triangle at B , AB = 6 cm. , BC = 8 cm. , if $\overline{BD}$ is a median of triangle ABC , then BD = ………. cm.
9	In Δ ABC , m (∠ C) = 60° , m (∠ B) = 90° , AC = 8 cm. , then BC = …………… cm.
10	In Δ ABC if m (∠ A) = 30° and m (∠ B) = 90° , then BC = ………. AC
11	If ABC : Is a right-angled at B , AB = $\frac{1}{2}$ AC , then m (∠ C) = ……….
12	If ABC is a right-angled triangle at B and AB = $\frac{1}{2}$ AC , then m (∠ A) = ……….
13	ABC is a right-angled triangle at B , if AC = 2 BC , then m (∠ C) = ……………°

14	In the opposite figure : The perimeter of Δ ABD = ……… cm.
15	**In the opposite figure :** D is the midpoint of $\overline{AC}$, m ($\angle$ E) = 30° , AC = 10 cm. **Find the length of :** $\overline{BE}$

[B] Essay Problems :

1	**In the opposite figure :** Δ ABC , AC = 8 cm. , m ($\angle$ BAC) = 60° , m ($\angle$ ABC) = 90° , D is the midpoint of $\overline{AC}$ **Find :** The perimeter of Δ ABD **2012 Exam (2) Question (5) (a)**
2	**In the opposite figure :** m ($\angle$ B) = 90° , m ($\angle$ C) = 30° , $\overline{BD}$ is a median , AB = 4 cm. , **Complete :** AC = ……… cm. , BD = ……… cm. , AD = ……… cm. **2012 Exam (10) Question (5) (a)**
3	**In the opposite figure :** Δ ABC in which m ($\angle$ B) = 90° , AC = 10 cm. , m ($\angle$ C) = 30° , EC = EB , AD = DC **Find with proof :** ❶ The perimeter of Δ ABD ❷ The length of $\overline{DF}$ **2012 Exam (7) Question (5) (a)**

4	**In the opposite figure :** m (∠ B) = 90° , m (∠ ACB) = 30° , E , F are midpoints of $\overline{AD}$, $\overline{DC}$ **Prove that :** AB = EF  **2015 Exam (3) Question (4) (b)**
5	**In the opposite figure :** m (∠ ABC) = m (∠ ADC) = 90° , m (∠ ACB) = 30° , and $\overline{DE}$ is a median of Δ ADC , If AB = 3 cm. **Find :** The length of $\overline{DE}$  **2012 Exam (8) Question (4) (a)**
6	**In the opposite figure :** m (∠ ABC) = m (∠ BDE) = 90° , m (∠ E) = 30° , D is the midpoint of $\overline{AC}$ **Prove that :** AC = BE  **2014 Exam (4) Question (4) (b)**

HOMEWORK

[A] Choose the correct answer:

1	The length of the hypotenous of the right-angled triangle = ... the length of the median which drawn from the vertex of the right-angle. (a) half (b) twice (c) third (d) quarter
2	The length of the median drawn from the vertex of right angle in the right-angled triangle = ... the length of the hypotenuse of the triangle. (a) 2 (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

3	In the right-angled triangle , the length of the median from the vertex of the right angle equal ……… the length of the hypotenuse. (a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) 2
4	In the right-angled triangle , the length of the median from the vertex of the right angle equals ……… the length of hypotenuse. (a) half (b) twice (c) third (d) forth
5	If Δ ABC is a right-angled at B , AB = 6 cm. , BC = 8 cm. , then the length of the medians drawn from B is ……… cm. (a) 10 (b) 8 (c) 6 (d) 5
6	In Δ ABC which is right at B , if AC = 20 cm. , then the length of the median of the triangle drawn from B equals ……… (a) 10 cm. (b) 8 cm. (c) 6 cm. (d) 5 cm.
7	In Δ ABC , m ($\angle$ B) = 90° , AC = 12 cm. and $\overline{BD}$ is a median in Δ ABC , then BD = ………… cm. (a) 12 (b) 6 (c) 24 (d) 10
8	The length of the side opposite to the angle of measure 30° in the right-angled ……… the length of the hypotenuse. (a) twice (b) half (c) square (d) equals
9	Triangle ABC : If m ($\angle$ A) = 30° , m ($\angle$ B) = 90° , then BC = ……… (a) $\frac{1}{2}$ AB (b) $\frac{1}{2}$ AC (c) 2 AB (d) 2 AC
10	In Δ ABC if : m ($\angle$ B) = 90° and m ($\angle$ A) = 60° , then AC = ………… AB (a) 2 (b) = (c) $\frac{1}{2}$ (d) $\frac{1}{3}$
11	Δ ABC : if m ($\angle$ A) = 30° and m ($\angle$ B) = 90° , then AC = ……… (a) $\frac{1}{2}$ BC (b) 2 BC (c) 2 AB (d) BC

12	In Δ ABC : m (∠ A) = 30° , m (∠ B) = 90° , AC = 10 cm. , then BC = ·········· cm. (a) 20 (b) 15 (c) 10 (d) 5
13	In Δ XYZ , if m (∠ Y) = 90° , m (∠ X) = 30° and XZ = 20 cm. , then ZY = ·············· cm. (a) 5 (b) 8 (c) 20 (d) 10
14	In the rectangle ACBD , if AC = 10 cm. , then BD = ·········· (a) 5 (b) 10 (c) 15 (d) 20
15	In the right-angled triangle , the length of the median from the vertex of the right angle equals ··········
16	If the length of the median drawn from a vertex of a triangle equals half the length of the opposite side to this vertex in length , then ··········
17	The length of the side opposite to the angle of measure 30° in the right-angled triangle equals ·········· the length of the hypotenuse.
18	The length of side opposite to the angle whose measure = 30° in the right-angled triangle = ··········
19	The length of the hypotenuse on the right-angled triangle equals ·········· the length of a side opposite to the angle of measure 30°
20	In Δ LMN : If m (∠ L) = 30° , m (∠ N) = 60° , NM = 4 cm. , then LN = ········· cm.
21	If ABC is a right-angled triangle at B , AB = 6 cm. , BC = 8 cm. , if $\overline{BD}$ is a median of triangle ABC , then BD = ········· cm.
22	In Δ ABC , m (∠ C) = 60° , m (∠ B) = 90° , AC = 8 cm. , then BC = ·············· cm.
23	In Δ ABC if m (∠ A) = 30° and m (∠ B) = 90° , then BC = ·········· AC

[B] Essay Problems :

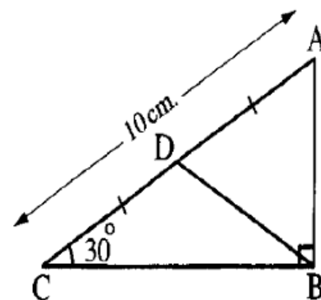
1

In the opposite figure :

m (∠ B) = 90° and m (∠ C) = 30° ,

AC = 10 cm.

Find : the lengths of $\overline{AB}$ and $\overline{BD}$



2015 Exam (12) Question (3) (b)

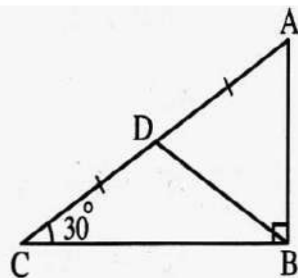
2

In the opposite figure :

m (∠ C) = 30°

Prove that :

AB = BD



2014 Exam (3) Question (4) (a)

3

In the opposite figure :

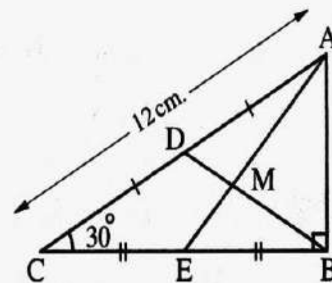
In Δ ABC : m (∠ B) = 90° , m (∠ C) = 30°

, D is the midpoint of $\overline{AC}$, E is the midpoint of $\overline{BC}$

, AC = 12 cm.

(1) **Find length of :** $\overline{BM}$

(2) **Find the perimeter of :** Δ ABC



2014 Exam (12) Question (5) (a)

4

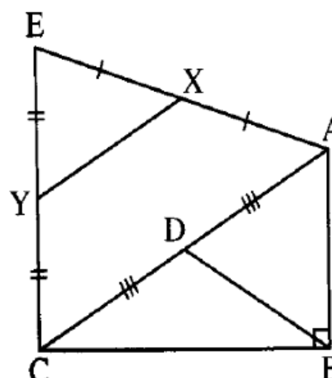
In the opposite figure :

X , Y , D are the midpoints of

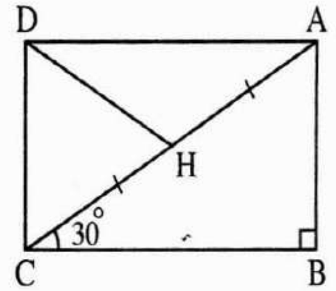
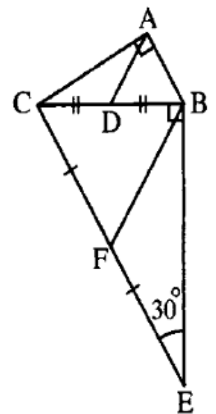
$\overline{EA}$, $\overline{EC}$ and $\overline{AC}$ respectively ,

m (∠ ABC) = 90°

Prove that : BD = YX



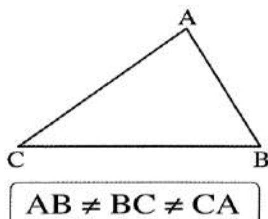
2015 Exam (6) Question (4) (a)

5	**In the opposite figure :** m (∠ B) = 90° , m (∠ ACB) = 30° , AB = DH where H is midpoint of $\overline{AC}$ **Prove that :** m (∠ ADC) = 90°  **2012 Exam (18) Question (5) (a)**
6	**In the opposite figure :** m (∠ BAC) = m (∠ CBE) = 90° , m (∠ BEC) = 30° , D and F are the midpoints of $\overline{BC}$ and $\overline{CE}$ respectively. **Prove that :** AD = $\frac{1}{2}$ BF  **2015 Exam (15) Question (4) (a)**

LESSON (3) The isosceles triangle

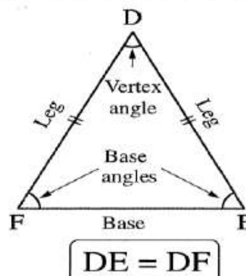
Triangles are classified according to the lengths of their sides into three types which are :

1 **Scalene triangle.**



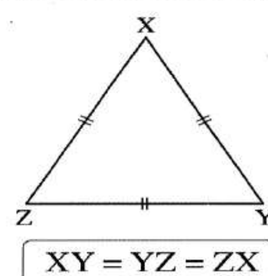
AB ≠ BC ≠ CA

2 **Isosceles triangle.**
(two sides are congruent).



DE = DF

3 **Equilateral triangle.**
(three sides are congruent).



XY = YZ = ZX

And in the following we will study the relations between the angles in the isosceles triangle and the equilateral triangle.

Mechanism : Isosceles Triangle :

The isosceles triangle theorem

Theorem 1

The base angles of the isosceles triangle are congruent.

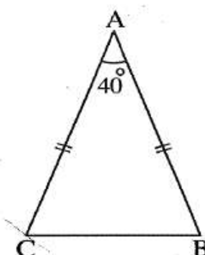
For example:

In the opposite figure :

If ABC is a triangle in which :

AB = AC , m (∠ A) = 40° ,

then m (∠ B) = m (∠ C) = $\frac{180° - 40°}{2}$ = 70°



Remarks

1 Both of the base angles in the isosceles triangle are acute.

2 The vertex angle in the isosceles triangle may be acute , right or obtuse angle.

Mechanism : Isosceles Triangle : Equilateral

Corollary

If the triangle is equilateral , then it is equiangular where each angle measure is 60°

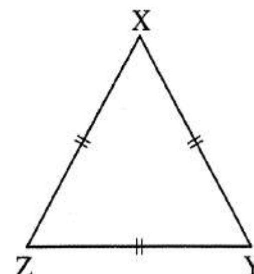
For example:

In the opposite figure :

If XYZ is a triangle in which

XY = YZ = ZX ,

then m (∠ X) = m (∠ Y) = m (∠ Z) = 60°



Examples on Part (1) : Isosceles Triangle

① **In the opposite figure :**

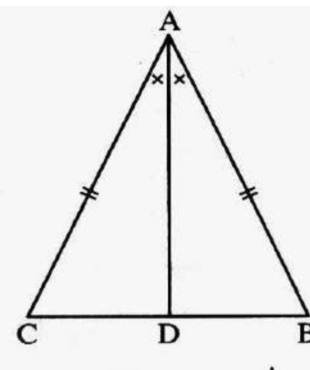
In Δ ABC :

AB = AC , $\overrightarrow{AD}$ bisects ∠ BAC

and BD = 3 cm.

Prove that : $\overline{AD} \perp \overline{BC}$

, then find the length of : $\overline{CB}$



Solution

In Δ ABC

∵ AB = AC

∵ AD bisects ∠BAC

∴ AD ⊥ = BC (First Req.)

∴ D is a midpoint of BC

∵ BD = 3 cm

∴ CD = BC = 3 cm

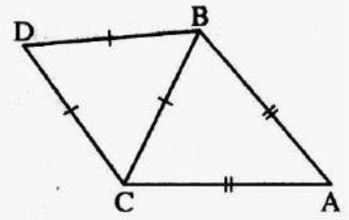
∴ CB = 3 × 2 = 6 cm (Second Req.)

② **In the opposite figure :**

m (∠ A) = 50° , AB = AC

and Δ DBC is an equilateral.

Find : m (∠ ABD)



Solution

In Δ ABC

∵ AB = AC

∵ m (∠A) = 50°

∴ m (∠ABC) = m (∠ACB)

∴ m (∠ABC) = (180 – 50) ÷ 2 = 65°

∵ Δ DBC is a equilateral

∴ m (∠DBC) = 60

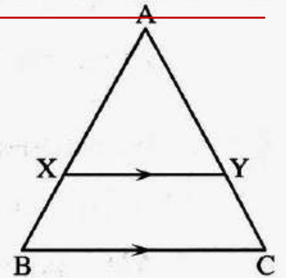
∴ m (∠ABD) = 65 + 60 = 125°

③ **In the opposite figure :**

If AB = AC ,

$\overline{XY}$ // $\overline{BC}$

Prove that : Δ AXY is an isosceles



Solution

In Δ ABC

∵ AB = AC

∴ m (∠B) = m (∠C)

∵ XY // BC , AC & AB are transversals

∴ m (∠B) = m (∠AXY) Corresponding

∴ m (∠C) = m (∠AYX) Corresponding

In Δ AXY

∴ m (∠AXY) = m (∠AYX)

∴ AX = AY

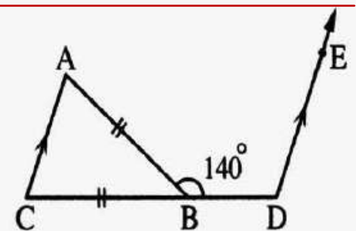
Δ AXY is an isosceles

④ **In the opposite figure :**

$\overrightarrow{CA}$ // $\overrightarrow{DE}$, m (∠ ABD) = 140°

AB = BC

Find : m (∠ EDB)



Solution

In Δ ABC

∴ m ∠A = m ∠C

∵ m∠ABD = m ∠A+ m ∠C (Exterior)

∵ m∠ABD = 140°

∴ m ∠A = m ∠C = 140 ÷ 2 = 70°

∵ AB = BC

∵ AC // DE , CD is a transversal

∴ m (∠C) + m (∠D) = 180 (Interior)

∴ m (∠D) = 180 – 70 = 110

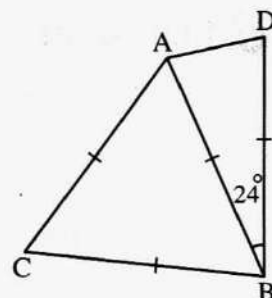
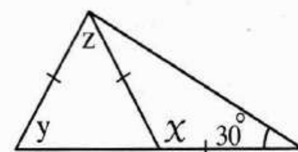
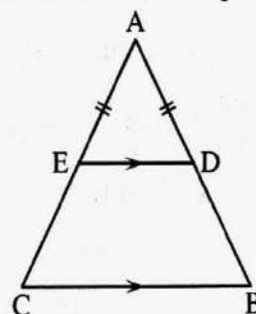
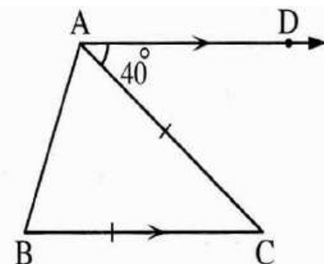
EXERCISES

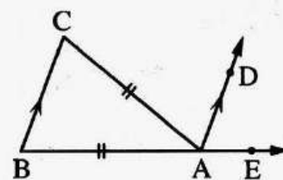
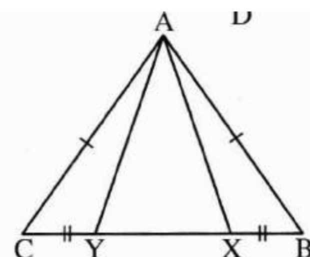
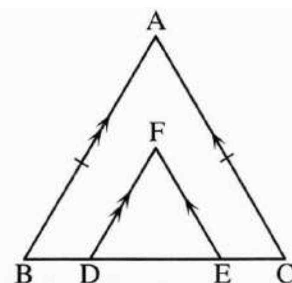
[A] Complete the Following :

1	The two base angles in an isosceles triangle are ……….
2	Δ ABC , AB = AC , m (∠ C) = 70° , then m (∠ A) = ……….
3	In the Δ ABC : AB = AC , m (∠ A) = 70° , then m (∠ C) = ……….°
4	The Δ ABC is an isosceles and right-angled triangle if m (∠ B) = 90° , then m (∠ A) = m (∠ C) = ……………°
5	In Δ ABC , if AB = AC and m (∠ A) = 80° , then m (∠ B) = m (∠ ……………) = ………°
6	In Δ ABC : if AB = AC , m (∠ B) = 60° , then the triangle is an ……….
7	In Δ ABC : If AB = AC and m (∠ A) = 2 m (∠ C) , then m (∠ B) = ………. °
8	The triangle whose side lengths 3 cm. , $(X + 1)$, and 6 cm. become isosceles triangle when X = ……….
9	The length of side opposite to the angle whose measure = 30° in the right-angled triangle = ……….
10	The length of the hypotenuse on the right-angled triangle equals ………. the length of a side opposite to the angle of measure 30°
11	In Δ LMN : If m (∠ L) = 30° , m (∠ N) = 60° , NM = 4 cm. , then LN = ……… cm.
12	If ABC is a right-angled triangle at B , AB = 6 cm. , BC = 8 cm. , if $\overline{BD}$ is a median of triangle ABC , then BD = ………. cm.
13	In Δ ABC , m (∠ C) = 60° , m (∠ B) = 90° , AC = 8 cm. , then BC = …………… cm.
14	In Δ ABC if m (∠ A) = 30° and m (∠ B) = 90° , then BC = ………. AC
15	If ABC : Is a right-angled at B , AB = $\frac{1}{2}$ AC , then m (∠ C) = ……….

16	If ABC is a right-angled triangle at B and AB = $\frac{1}{2}$ AC , then m (∠ A) = ………
17	ABC is a right-angled triangle at B , if AC = 2 BC , then m (∠ C) = ……………°

[B] Essay Problems :

1	**In the opposite figure :** ACBD is a quadrilateral in which : AB = BC = CA = BD , m (∠ ABD) = 24° **Find :** m (∠ CAD)  **2014 Exam (9) Question (4) (b)**
2	**In the opposite figure complete :** x = ……… ° , y = ……… ° , z = ……… °  **2012 Exam (7) Question (4) (a)**
3	**Find the measures of the angles in : Δ ABC** **In the opposite figure :** $\overline{DE}$ // $\overline{BC}$, AD = AE **Prove that :** AB = AC  **2014 Exam (13) Question (5) (a)**
4	**In the opposite figure :** ABC is a triangle , AC = BC , $\overrightarrow{AD}$ // $\overline{BC}$, m (∠ DAC) = 40° **Find :** The measure of angles in the Δ ABC  **2012 Exam (13) Question (5) (b)**

5	**In the opposite figure :** AB = AC , $\overrightarrow{AD}$ // $\overline{BC}$ **Prove that :** $\overrightarrow{AD}$ bisects ∠ CAE  **2014 Exam (12) Question (4) (a)**
6	**In the opposite figure :** In Δ ABC , AB = AC , BX = CY **Prove that :** AX = AY  **2012 Exam (13) Question (4) (b)**
7	**In the opposite figure :** D ∈ $\overline{BC}$, E ∈ $\overline{BC}$, $\overline{AB}$ // $\overline{FD}$ and $\overline{AC}$ // $\overline{FE}$, if AB = AC **Prove that :** FDE is an isosceles triangle.  **2012 Exam (6) Question (4) (a)**

HOMEWORK

[A] Choose the correct answer:

1	In any isosceles triangle , the type of the base angles is ········· (a) acute. (b) right. (c) obtuse. (d) reflex.
2	The base angles of the isosceles triangle are ··············· (a) congruent. (b) alternate. (c) corresponding. (d) supplementary.
3	If measure of one of the two base angles of the isosceles triangle equals 40° then the measure of the vertex angle = ··············° (a) 40 (b) 100 (c) 80 (d) 50

4	In Δ ABC : AB = AC , m (∠ B) = 50° , then m (∠ A) = ...° (a) 65 (b) 80 (c) 50 (d) 100
5	An isosceles triangle , one of its base angles has measure 50° , then the measure of the vertex angle = ... (a) 50° (b) 60° (c) 70° (d) 80°
6	In the isosceles triangle , if the measure of one of the two base angle is 70° , then the measure of its vertex angle is .... (a) 70° (b) 110° (c) 20° (d) 40°
7	The measure of one angle of the two base angles of the isosceles = 75° , then the measure of the vertex angle = ... (a) 50° (b) 75° (c) 30° (d) 105°
8	In a triangle ABC : If AB = AC and m (∠ A) = 40° , then m (∠ C) = ... (a) 40° (b) 70° (c) 140° (d) 50°
9	In Δ ABC , AB = AC , m (∠ A) = 50° , then m (∠ B) = ... (a) 50° (b) 65° (c) 130° (d) 100°
10	If the measure of an angle of the isosceles triangle is 100° , then the measure of one of the other angles = ... (a) 50° (b) 80° (c) 40° (d) 100°
11	Δ XYZ is an isosceles triangle in which m (∠ X) = 100° , then m (∠ Y) = ...° (a) 100 (b) 80 (c) 60 (d) 40
12	ABC is a triangle in which AB = AC and m (∠ A) = 110°, then m (∠ B) = ... (a) 70° (b) 55° (c) 35° (d) 110°
13	If the measure of an angle of the isosceles triangles is 120° , then the measure of one of the other angles = ... (a) 60° (b) 30° (c) 40° (d) 45°
14	ABC is isosceles triangle m (∠ C) = 130° , then m (∠ B) = ...° (a) 130 (b) 50 (c) 25 (d) 60

15	The triangle whose sides lengths are 2 cm. , $(X+1)$ cm and 5 cm. becomes an isosceles triangle when $X =$ ········· cm. (a) 1 (b) 2 (c) 3 (d) 4
16	The triangle whose sides lengths are 3 cm. , $(X+5)$ and 9 becomes an isosceles if $X =$ ········· cm. (a) 3 (b) 4 (c) 5 (d) 6
17	Triangle whose sides lengths are 2 cm. , $(X-2)$ cm. , 5 cm. becomes isosceles triangle when $X =$ ········· cm. (a) 3 (b) 4 (c) 5 (d) 7
18	**In the opposite figure :** ABC is a triangle in which : m (∠ B) = m (∠ C) , then $X =$ ········· (a) 1 (b) 2 (c) 3 (d) 4 A, B, C, $X+1$, $2X-1$, $2X$
19	ABCD is a parallelogram : DE = DC , m (∠ A) = 50° , then m (∠ EDC) = ········· (a) 50° (b) 60° (c) 70° (d) 80° D, A, 50°, C, E, B

[B] Essay Problems :

1	**In the opposite figure :** AB = AD , m (∠ A) = 30° , CB = BD = CD **Find :** m (∠ CBA) B, C, 30°, A, D **2014 Exam (4) Question (5) (b)**
2	**In the opposite figure :** AB = BC = AC = DC **Prove that :** m (∠ BAD) = 90° A, D, C, B **2012 Exam (10) Question (4) (a)**

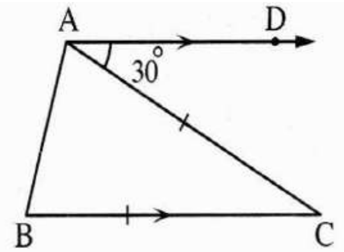
3

In the opposite figure :

ABC is a triangle in which : AC = BC ,

$\overline{AD} \parallel \overline{BC}$, m (∠ DAC) = 30°

Find : m (∠ ABC)



2012 Exam (4) Question (4) (b)

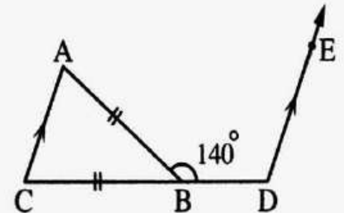
4

In the opposite figure :

$\overline{CA} \parallel \overrightarrow{DE}$, m (∠ ABD) = 140°

AB = BC

Find : m (∠ EDB)



2014 Exam (15) Question (3) (b)

5

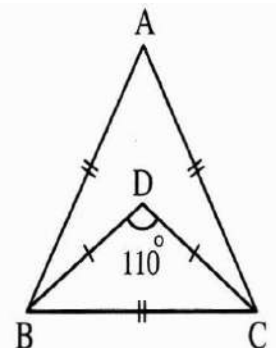
Find : The length of AM , MC and AC

In the opposite figure :

ABC is an equilateral triangle ,

DB = DC , m (∠ D) = 110°

Find with proof : m (∠ DBC) and m (∠ DBA)



2012 Exam (3) Question (3) (b)

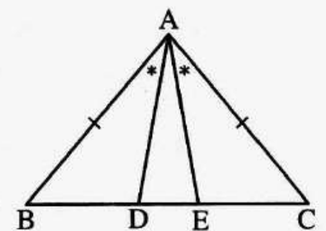
6

In the opposite figure :

AB = AC , m (∠ BAD) = m (∠ CAE)

Prove that : (1) AD = AE

(2) m (∠ AED) = m (∠ ADE)



2014 Exam (12) Question (3) (b)

7

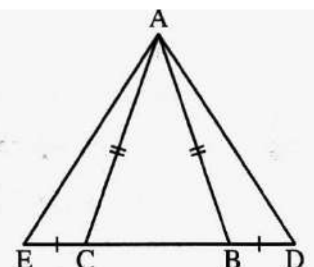
Find the length of : BD , BM and AB

] **In the opposite figure :**

ADE is a triangle , B ∈ $\overline{DE}$, C ∈ $\overline{DE}$

, BD = CE , AB = AC

Prove that : AD = AE



2014 Exam (9) Question (3) (b)

LESSON (4) The conversemof the isosceles triangle

Mechanism : To prove it is Isosceles Triangle :

Theorem 2

If two angles of a triangle are congruent , then the two sides opposite to these two angles are congruent and the triangle is isosceles.

Remark

The isosceles triangle in which the measure of one of its angles = 60° is an equilateral triangle.

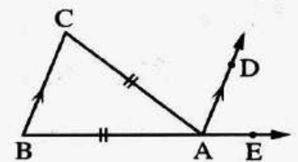
Examples on Part (1) : Isosceles Triangle

(5)] **In the opposite figure :**

AB = AC ,

$\overrightarrow{AD}$ // $\overline{BC}$

Prove that : $\overrightarrow{AD}$ bisects ∠ CAE



Solution

In Δ ABC

∵ AB = AC

∴ m ∠B = m ∠C

∵ AC // DE , AB & AC are transversals

∴ m ∠B = m ∠DAE (Corresponding)

∴ m ∠C = m ∠CAD (Alternate)

∴ m ∠DAE = m ∠CAD

AD bisects m ∠CAE

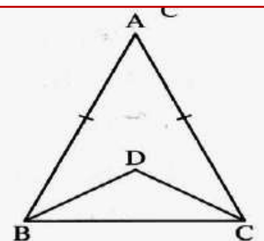
(6) **In the opposite figure :**

ABC is a triangle in which AB = AC ,

$\overrightarrow{BD}$ bisects ∠ ABC , $\overrightarrow{CD}$ bisects ∠ ACB

Prove that :

Δ DBC is an isosceles triangle.



Solution

In Δ ABC

∵ AB = AC

∴ m ∠B = m ∠C

∵ BD bisects m ∠ABC

∵ CD bisects m ∠ACB

∴ m∠DBC = $\frac{1}{2}$m ∠ABC

∴ m∠DCB = $\frac{1}{2}$m ∠ACB

∴ m∠DBC = m∠DCB

∴ Δ DBC is an isosceles

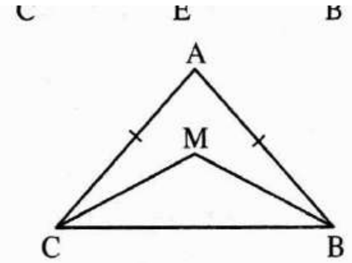
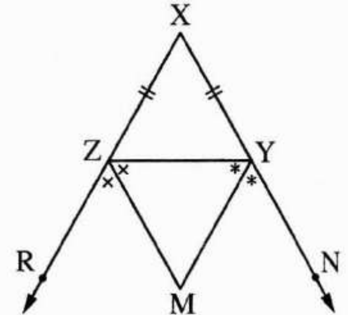
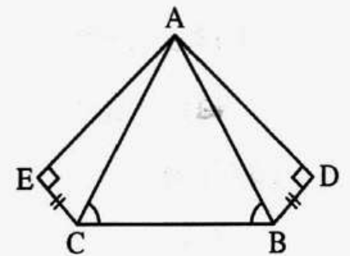
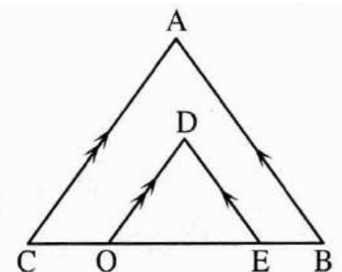
EXERCISES

[A] Complete the Following :

1	If angles of any triangle are equal in measures , then the triangle is ....
2	If the angles of a triangle are congruent , then the triangle is ....
3	The measure of the exterior angle of equilateral triangle = ...°
4	The measure of any exterior angle of the triangle is greater than ....
5	If the measure of one of the angles of the right-angled triangle is 45° , then the triangle is ....
6	In an isosceles triangle , if any angle has a measure of 60° , the triangle is ....
7	In Δ ABC if : $\overline{AB} \perp \overline{BC}$ and AB = BC , then m (∠ A) = ...°

[B] Essay Problems :

1	**In the opposite figure :** $\overrightarrow{AD}$ bisects ∠ BAC , m (∠ B) = 30° , m (∠ C) = 70° **Prove that :** Δ ADC is isosceles triangle. **2014 Exam (15) Question (5) (b)**
2	ABC is a triangle in which : m (∠ A) = 50° and m (∠ C) = 80° **Prove that :** this triangle ABC is an isosceles triangle. **2** **015 Exam (1) Question (4) (a)**

3	**In the opposite figure :** AB = AC , $\overrightarrow{BM}$ and $\overrightarrow{CM}$ bisect the angles (∠ B) , (∠ C) **Prove that :** MB = MC  **2012 Exam (1) Question (3) (b)**
4	**In the given figure :** XY = XZ , $\overrightarrow{YM}$ bisects (∠ ZYN) , $\overrightarrow{ZM}$ bisects (∠ YZR) , then **Prove that :** ❶ Δ MYZ is isosceles. ❷ $\overleftrightarrow{XM}$ is the axis of symmetry of $\overline{ZY}$  **2012 Exam (5) Question (3) (a)**
5	**In the opposite figure :** BD = CE , m (∠ ABC) = m (∠ ACB) , m (∠ D) = m (∠ E) = 90° **Prove that :** m (∠ DAB) = m (∠ CAE)  **2014 Exam (6) Question (3) (a)**
6	**In the opposite figure :** AB = AC , $\overline{DE}$ // $\overline{AB}$ and $\overline{AC}$ // $\overline{DO}$ **Prove that :** ❶ DE = DO ❷ m (∠ A) = m (∠ D)  **2012 Exam (16) Question (4) (a)**

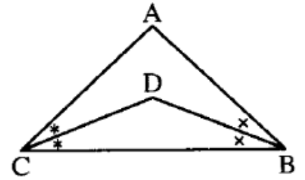
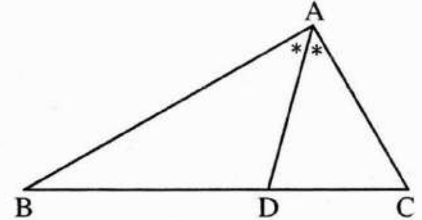
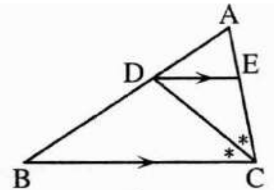
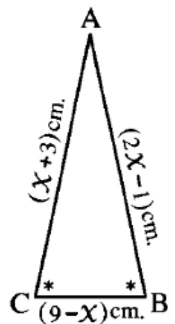
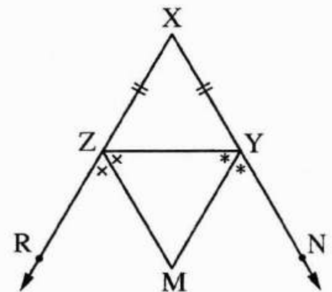
HOMEWORK

[A] Choose the correct answer:

1	The measure of exterior angle of an equilateral triangle = ………. (a) 30° (b) 60° (c) 120° (d) 180°
2	In Δ XYZ : if XY = XZ , then the exterior angle at the vertex Z is ………. (a) acute. (b) obtuse. (c) right. (d) reflex.
3	In Δ ABC : if AB = AC and m (∠ A) = 60° , if its perimeter is 18 cm. , then BC = ………. cm. (a) 18 (b) 6 (c) 3 (d) 60
4	Δ ABC , AB = AC , D is the midpoint of $\overline{BC}$, then $\overline{AD}$ is ………. (a) median. (b) altitude. (c) bisector of the vertex angle. (d) all the previous.

[B] Essay Problems :

1	**In the opposite figure :** AD = AC , m (∠ DAB) = 30° , m (∠ ABD) = 40° **Prove that :** AB = CB 2 **015 Exam (6) Question (5) (a)**
2	**In the opposite figure :** ABC is a triangle in which AB = AC , X ∈ $\overline{AB}$, Y ∈ $\overline{AC}$ and $\overline{XY}$ // $\overline{BC}$ **Prove that :** the triangle AXY is isosceles triangle. **2012 Exam (15) Question (3) (a)**

3	**In the opposite figure :** AB = AC , $\overrightarrow{BD}$ bisects ∠ B and $\overrightarrow{CD}$ bisects ∠ C **Prove that :** Δ DBC is an isosceles triangle  **2015 Exam (15) Question (3) (b)**
4	ABC is a triangle in which : AB > AC and ∠ BAC is bisected by $\overrightarrow{AD}$ which intersects $\overline{BC}$ at D **Prove that :** Δ ABD is an obtuse-angled triangle.  **2012 Exam (20) Question (4) (a)**
5	**In the opposite figure :** $\overrightarrow{CD}$ bisects ∠ ACB , $\overline{DE}$ // $\overline{CB}$ **Prove that :** Δ ECD is an isosceles triangle.  **2012 Exam (12) Question (3) (b)**
6	**In the opposite figure :** m (∠ B) = m (∠ C) , AB = (2 x – 1) cm. AC = (x + 3) cm. , BC = (9 – x) cm. Find with proof the perimeter of Δ ABC  **2015 Exam (3) Question (5) (b)**
7	**In the given figure :** XY = XZ , $\overrightarrow{YM}$ bisects (∠ ZYN) , $\overrightarrow{ZM}$ bisects (∠ YZR) , then **Prove that :** ❶ Δ MYZ is isosceles. ❷ $\overleftrightarrow{XM}$ is the axis of symmetry of $\overline{ZY}$  **2012 Exam (5) Question (3) (a)**

LESSON (5) conversemof the isosceles triangle theorems

Mechanism : Isosceles Triangle : Median

Corollary 1

The median of an isosceles triangle from the vertex angle bisects it and is perpendicular to the base.

In the opposite figure :

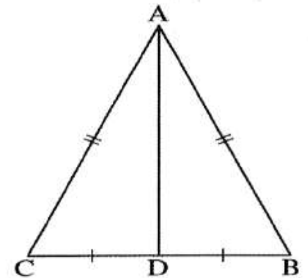
ABC is a triangle in which AB = AC and

$\overline{AD}$ is a median , then :

1 $\overrightarrow{AD}$ bisects $\angle$ BAC

i.e. m ($\angle$ BAD) = m ($\angle$ CAD)

2 $\overline{AD} \perp \overline{BC}$



Mechanism : Isosceles Triangle : Vertex Bisector

Corollary 2

The bisector of the vertex angle of an isosceles triangle bisects the base and is perpendicular to it.

n the opposite figure :

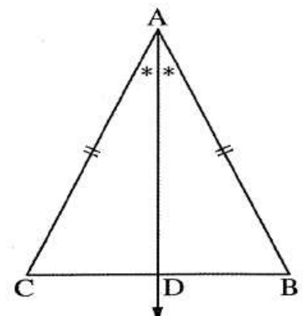
ABC is a triangle in which AB = AC and

$\overrightarrow{AD}$ bisects $\angle$ BAC , then :

1 D is the midpoint of $\overline{BC}$

i.e. BD = CD

2 $\overline{AD} \perp \overline{BC}$



Mechanism : Isosceles Triangle : Perpendicular

Corollary 3

The straight line drawn passing through the vertex angle of an isosceles triangle perpendicular to the base bisects each of the base and the vertex angle.

In the opposite figure :

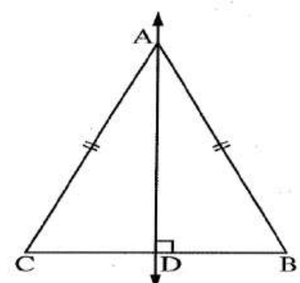
ABC is a triangle in which AB = AC and

$\overleftrightarrow{AD} \perp \overline{BC}$, then :

1 D is the midpoint of $\overline{BC}$

i.e. BD = CD

2 m ($\angle$ BAD) = m ($\angle$ CAD)



Notice that :

The previous three corollaries can be proved using the congruence of Δ ABD and Δ ACD

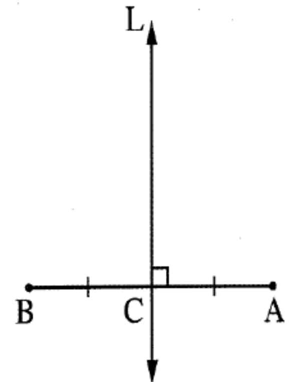
Mechanism: Axis of symmetry of line segment (1)

Definition

The straight line perpendicular to a line segment at its middle is called the axis of symmetry for that line segment , in brief it is known as the axis of a line segment.

In the opposite figure :

If the straight line L $\perp$ $\overline{AB}$ and C $\in$ the straight line L where C is the midpoint of $\overline{AB}$, then the straight line L is called the axis of $\overline{AB}$



Mechanism: Axis of symmetry of line segment (2)

Property

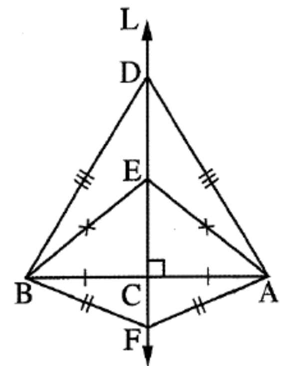
Any point on the axis of symmetry of a line segment is at equal distances from its terminals (end points).

In the opposite figure :

If the straight line L is the axis of $\overline{AB}$,

D $\in$ L , E $\in$ L and F $\in$ L , then

DA = DB , EA = EB and FA = FB

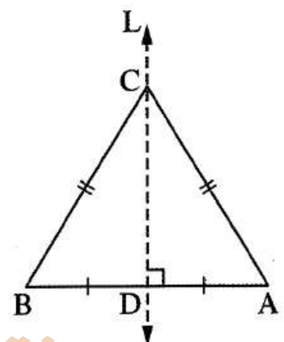


The converse of the previous property is true

i.e. If a point is at equal distances from the two terminals of a line segment , then this point lies on the axis of this line segment.

In the opposite figure :

If C is a point such that CA = CB , then the point C lies on the axis of $\overline{AB}$



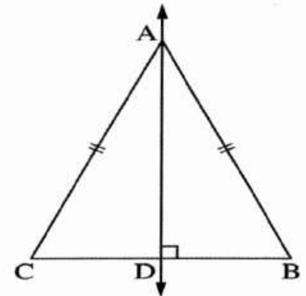
Mechanism: Axis of symmetry of Isosceles Triangle

The isosceles triangle has one axis of symmetry.

It is the straight line drawn from the vertex angle perpendicular to its base.

For example:

If ABC is an isosceles triangle where $AB = AC$ and $\overleftrightarrow{AD} \perp \overline{BC}$, then $\overleftrightarrow{AD}$ is called the axis of symmetry of the isosceles triangle ABC



Mechanism : Axis of Symmetry of Equilateral Triangle

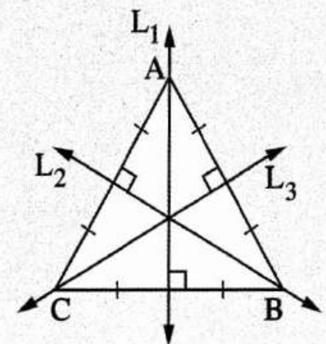
Remarks

1. The equilateral triangle has three axes of symmetry , they are the three perpendiculars drawn from its vertices to the opposite sides.

In the opposite figure :

The straight lines L_1 , L_2 and L_3 are the axes of symmetry of the equilateral triangle ABC

2. The scalene triangle has no axes of symmetry.



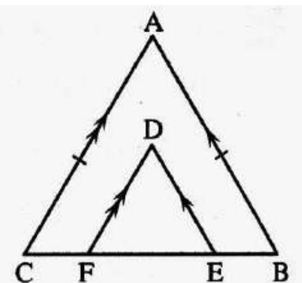
Examples on Part (1) : Isosceles Triangle

Find the perimeter of the figure : ADME

⑦ **In the opposite figure :**

$AB = AC$, $\overline{DE} // \overline{AB}$, $\overline{DF} // \overline{AC}$

Prove that : $DE = DF$



Solution

In Δ ABC

∴ m ∠B = m ∠C

∵ AC // DF , CF is a transversal

∵ AB // DE , EB is a transversal

∴ m ∠B = m ∠DEF

(Corresponding)

∵ AB = AC

∴ m ∠C = m ∠DFE

(Corresponding)

∵ In Δ DEF

∴ m ∠DEF = m ∠DFE

∴ DE = DF

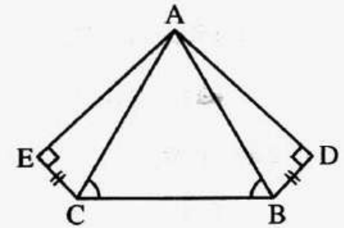
(8) **In the opposite figure :**

BD = CE

, m (∠ ABC) = m (∠ ACB)

, m (∠ D) = m (∠ E) = 90°

Prove that : m (∠ DAB) = m (∠ CAE)



Solution

In Δ ABC

∵ m ∠ABC = m ∠ACB

∴ AB = AC

In Δ Δ ABD , ACE

1) AB = AC

2) BD = CE

3) m∠D = m∠E = 90°

∴ Δ ABD ≡ Δ ACE

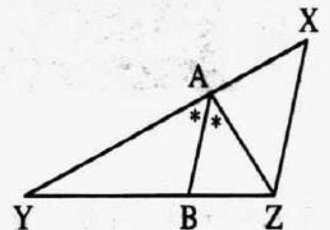
∴ m ∠DAB = m ∠ CAE

(9) **In the opposite figure :**

$\overrightarrow{AB}$ bisects angle YAZ

, $\overline{AB}$ // $\overline{XZ}$

Prove that : Δ AXZ isosceles triangle.



Solution

In Δ XYZ

∵ AB // XZ , AZ is a transversal

∴ m ∠BAZ = m ∠AZX

(Alternate)

∵ AB // XZ , AX is a transversal

∴ m ∠X = m ∠BAY

(Corresponding)

∵ AB bisects angle YAZ

∴ m ∠YAB = m ∠ZAB

∴ m ∠X = m ∠ AZX

∴ AZ = AX

Δ XYZ is isosceles triangle

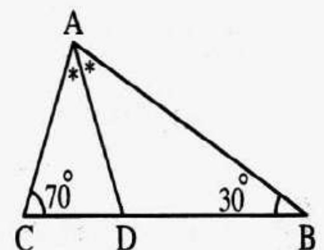
(10) **In the opposite figure :**

$\overrightarrow{AD}$ bisects ∠ BAC

, m (∠ B) = 30°

, m (∠ C) = 70°

Prove that : Δ ADC is isosceles triangle.



Solution

In Δ ABC
∵ m ∠B = 30°
∵ m ∠C = 70°
∴ m ∠BAC = 180 – 30 – 70 = 80°
∵ AD bisects angle BAC
∴ m ∠BAD = m ∠CAD = 80 ÷ 2 = 40°

∴ In Δ ADC
∴ m ∠ ADC = 180 – 70 – 40 = 70°
∴ m ∠ ADC = m ∠ ACD = 70°
∴ AD = AC
Δ ADC is isosceles triangle

EXERCISES

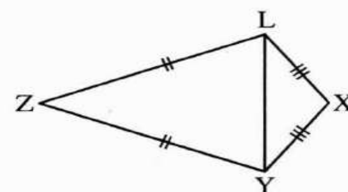
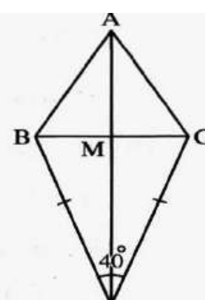
[A] Complete the Following :

1	The ray drawn from the vertex of the isosceles triangle passing through the midpoint of the base is ……………
2	The median of an isosceles triangle drawn from the vertex bisects ………. and is perpendicular to ……….
3	The bisector of the vertex angle of an isosceles triangle ………. and ……….
4	In Δ XYZ : If XY = XZ , $\overrightarrow{XL} \perp \overline{YZ}$, then $\overleftrightarrow{XL}$ bisects each of ………. and ……….
5	The straight line perpendicular to the midpoint of a line segment is called ……….
6	In the isosceles triangle if the measure of any angle is 60° , then the number of axis of symmetry ……….
7	The number of axes of symmetry of the isosceles triangle equal ……….
8	The number of symmetrical line in an scalene triangle = ……….
9	The number of the axes of symmetry in an equilateral triangle = ……….
10	The number of axes of symmetry of the triangle in which the measures of two angles are 50° , 70° = ……….
11	In Δ ABC : If AB = AC , then the point A lies on the axis of symmetry of ……….

12	If D is the midpoint of $\overline{AB}$ and $\overleftrightarrow{CD} \perp \overline{AB}$, then CA = ……………
13	The axis of symmetry of the line segment is the straight line which ……….
14	Any point on the axis symmetry of a line segment is at two equal distance from ……….
15	If the point A ∈ the axis of symmetry of $\overline{BC}$, then AB = ………
16	The axis of symmetry of isosceles triangle is ……….

[B] Essay Problems :

1	**In the opposite figure :** In Δ ABC , AB = AC , $\overline{AD} \perp \overline{BC}$, AB = 13 cm. and BD = 5 cm. **Find :** ❶ The length of $\overline{BC}$ ❷ The area of Δ ABC (Figure labels: A, C, D, B, 13 cm., 5 cm.) **2012 Exam (11) Question (5) (b)**
2	**In the opposite figure :** $\overrightarrow{AD}$ bisects ∠ BAC , AB = AC = 17 cm. , and BC = 16 cm. **Prove that :** m (∠ ADB) = 90° , then find the length of : $\overline{AD}$ and the area of Δ ABC (Figure labels: A, B, D, C, 17 cm., 17 cm., 16 cm.) **2015 Exam (2) Question (4) (a)**
3	**In the opposite figure :** ABC is a triangle in which : AB = AC , $\overline{AD} \perp \overline{BC}$ m (∠ BAC) = 100° and BD = 3 cm. **Find :** (1) m (∠ BAD) (2) The length of $\overline{CB}$ (Figure labels: A, C, D, B) **2015 Exam (13) Question (4) (a)**

4	**In the opposite figure :** XL = XY , ZL = ZY , M is the midpoint of $\overline{LY}$ **Prove that :** X , M , Z are on the same straight line.  **2012 Exam (2) Question (5) (b)**
5	**In the opposite figure :** DB = DC , m (∠ D) = 40° , Δ ABC is an equilateral triangle , if M is the midpoint of $\overline{BC}$ (1) **Find :** m (∠ ABD) (2) **Prove that :** A , M and D are on the same straight line.  **2014 Exam (3) Question (5) (a)**

HOMEWORK

[A] Choose the correct answer:

1	The axis of symmetry of a line segment is the straight line which is .... (a) Perpendicular to it. (b) its bisector. (c) parallel to it. (d) the perpendicular bisector.
2	If A ∈ the axis of symmetry of $\overline{BC}$, then $\overline{AB}$ .... $\overline{AC}$ (a) ⊥ (b) ≡ (c) // (d) =
3	If A lies on the axis of symmetry of $\overline{XY}$ then AX .... AY (a) // (b) ⊥ (c) = (d) ≠
4	The number of axis of symmetry in the scalene triangle is .... (a) 1 (b) zero (c) 3 (d) 4
5	The number of axes of symmetry in the isosceles triangle is .... (a) 1 (b) 2 (c) 3 (d) zero

6	The isosceles triangle has ·············· axis (axes) of symmetry. (a) no (b) two (c) only one (d) three
7	The number of axes of symmetry in the equilateral triangle is ·········· (a) 0 (b) 2 (c) 3 (d) 1
8	The equilateral triangle has ········· axes of symmetry. (a) one (b) two (c) three (d) otherwise
9	The triangle which has no axes of symmetry is ·········· triangles. (a) scalene (b) isosceles (c) equilateral (d) otherwise
10	If Δ ABC has one axes of symmetry and m ($\angle$ ABC) = 140° , then m ($\angle$ A) = ········· (a) 30° (b) 20° (c) 40° (d) 60°
11	The triangle which has three axes of symmetry is ········· triangle. (a) scalene (b) isosceles (c) right-angled (d) equilateral
12	Δ ABC in which m ($\angle$ A) = m ($\angle$ B) = 65° , then it has ·············· axis (axes) of symmetry. (a) 1 (b) 2 (c) 3 (d) zero
13	In Δ ABC if : m ($\angle$ A) = 40° and m ($\angle$ B) = 70° , then Δ ABC has ·············· axis (axes) of symmetry. (a) 3 (b) 1 (c) 2 (d) zero
14	The quadrilateral ABCD in which $\overleftrightarrow{BD}$ is an axis of symmetry of $\overline{AC}$ may by ········· (a) a rhombus (b) a rectangle (c) a parallelogram (d) a trapezium

[B] Essay Problems :

1

In the opposite figure :

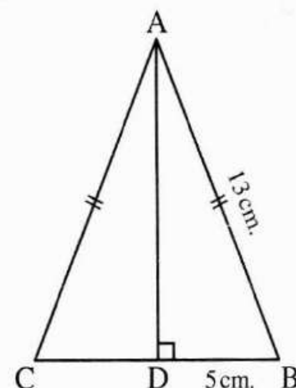
In Δ ABC , AB = AC ,

$\overline{AD} \perp \overline{BC}$,

AB = 13 cm. and BD = 5 cm.

Find : (1) The length of $\overline{BC}$

(2) The area of Δ ABC



2012 Exam (11) Question (5) (b)

2

In the opposite figure :

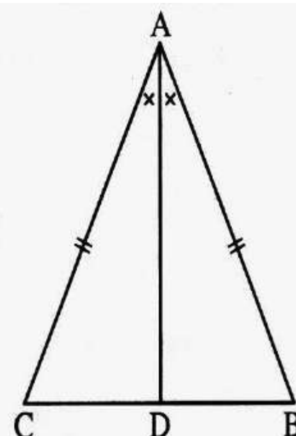
In Δ ABC :

AB = AC , $\overrightarrow{AD}$ bisects $\angle$ BAC

and BD = 3 cm.

Prove that : $\overline{AD} \perp \overline{BC}$

, then find the length of : $\overline{CB}$



2014 Exam (1) Question (5) (a)

3

In the opposite figure :

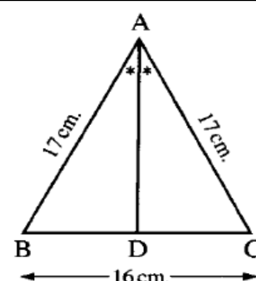
$\overrightarrow{AD}$ bisects $\angle$ BAC ,

AB = AC = 17 cm. ,

and BC = 16 cm.

Prove that : m ($\angle$ ADB) = 90° ,

then find the length of : $\overline{AD}$ and the area of Δ ABC



2015 Exam (2) Question (4) (a)

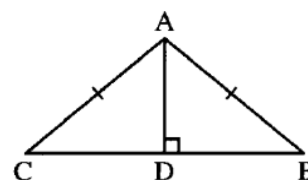
4

In the opposite figure :

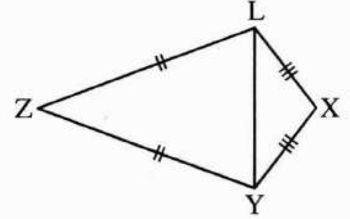
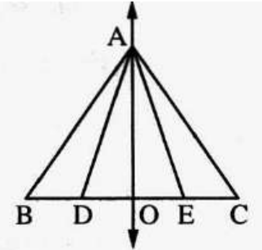
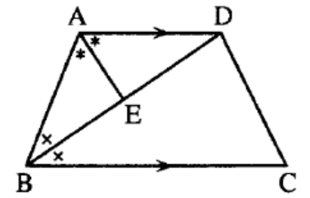
ABC is a triangle in which : AB = AC , $\overline{AD} \perp \overline{BC}$

m ($\angle$ BAC) = 100° and BD = 3 cm.

Find : (1) m ($\angle$ BAD) (2) The length of $\overline{CB}$



2015 Exam (13) Question (4) (a)

5	**In the opposite figure :** XL = XY , ZL = ZY , M is the midpoint of $\overline{LY}$ **Prove that :** X , M , Z are on the same straight line.  **2012 Exam (2) Question (5) (b)**
6	**In the opposite figure :** The two triangles ABC , AED have the same line of symmetry $\overleftrightarrow{AO}$ What type of each them according to its sides ? **Prove that :** BD = EC  **2014 Exam (10) Question (4) (b)**
7	**In the opposite figure :** ABCD is a quadrilateral in which $\overline{AD}$ // $\overline{BC}$, $\overrightarrow{BD}$ bisects ∠ ABC , $\overrightarrow{AE}$ bisects ∠ BAD **Prove that :** (1) AB = AD (2) $\overline{AE} \perp \overline{BD}$ (3) BE = ED  **2015 Exam (11) Question (5) (a)**